2026 Online Physics Olympiad:

Last updated: 22nd August 03:35 UTC

Sponsors

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Corrections (all times are in UTC)

- Q8: Cleared up wording (21/08 04:40 and 04:52)
- Q3: improved wording (21/08 08:49)
- Q19: Improved wording (21/08 09:42)
- Q16: Attempts reset (21/08 22:00), Error bound increased (22/8 02:30)
- Q1: Improved wording (22/08 03:35)

Instructions

Format. The OPhO is a team-based competition (3 members at most per team). Unlike previous years, the 2026 OPhO consists of a single round held from August 21, 12 am (UTC), to August 23, 11:59 pm (UTC). The contest is made up of three parts, all running concurrently over the full contest window:

- **Short Answer** — problems requiring numerical answers, submitted on the OPhO portal;
- **Long Answer** — problems requiring full written solutions, submitted on the OPhO portal;
- **Simulation** — computational problems requiring teams to run a provided executable locally, with solutions submitted on the OPhO portal.

All teams have access to all three parts for the entire duration of the contest. Submissions are made on the OPhO portal https://opho.physoly.org/ using the team login, and may be made at any time within the contest window.

Short Answer. Teams submit numerical answers on the portal, with up to three attempts per problem. *Every* answer submission must be accompanied by a file upload, on the portal, of the working used to obtain that answer. This working will not be graded, but submissions without it will not be accepted. The base score for each problem depends on the number of teams that successfully answer it, each incorrect attempt multiplies the base score by a factor of 0.8, and there is no time factor. The score for an individual problem follows:

$$w(N) = (0.8)^i \left[e^{-\frac{x(N)}{200}} + e^{-\frac{x(N)^2}{100}} + 1 \right],$$

where $i \in \{0, 1, 2\}$ represents whether a team got it on the first, second, or third try, N represents the number of teams that answer the problem correctly, and $x(N)$ is the sum of $(0.8)^i$ across all teams that have solved the problem. The base score is independent of problem number, but expect questions to be ordered roughly by difficulty. A team's raw Short Answer total is the sum of these values, and all raw totals are then scaled linearly such that the top team receives 55 points.

Long Answer. Teams submit full solutions on the portal. Rubrics are predetermined, and solutions will be reviewed to determine the proper number of points to give. The points breakdown is given per problem, for a total of 35 points. It is recommended that participants write their solutions in LaTeX. Handwritten solutions (or a combination of both) are accepted too; if participants have more than one photo of a handwritten solution (`jpg`, `png`, etc.), it is required that they be organized in the correct order in a `pdf` before submitting.

Simulation. Teams will need to be able to run a provided executable locally. Solutions are submitted on the portal and will be graded against predetermined rubrics. The points breakdown is given per problem, for a total of 10 points.

Scoring. A team's cumulative score is the sum of its scaled Short Answer score (out of 55 points), Long Answer score (out of 35 points), and Simulation score (out of 10 points), for a maximum score of 100 points.

Permitted Resources. Use of the internet is allowed, including textbooks, published papers, calculators, and computer algebra systems. However, the use of AI tools of any kind is strictly prohibited. This includes, but is not limited to, large language models (e.g. ChatGPT, Claude, Gemini), AI coding assistants, AI-generated search summaries, and any other AI-powered assistance.

Dishonest Behavior. Please refrain from cheating in any form. This includes, but is not limited to, creating alternate accounts to gain additional attempts, sharing answers with other teams, posting problems online, utilizing AI tools as described above, or employing any other method to gain an unfair advantage.

We have previously caught numerous teams engaging in such practices, and pursuing dishonest tactics provides no genuine sense of security. Any teams found to be acting maliciously will be notified via email, along with the opportunity to appeal. Failing to provide a valid appeal will result in a ban from participating in any future Physoly competitions. Our utmost priority is to ensure an enjoyable contest experience for all participants, and we remain committed to preventing any disruption to the fun for others.

Awards and Results. The top competitors will be recognized on the website. No cash prizes will be awarded this year.

List of Constants

- Proton mass, $m_p = 1.67 \cdot 10^{-27}$ kg
- Neutron mass, $m_n = 1.67 \cdot 10^{-27}$ kg
- Electron mass, $m_e = 9.11 \cdot 10^{-31}$ kg
- Avogadro's constant, $N_0 = 6.02 \cdot 10^{23}$ mol^{-1}
- Universal gas constant, $R = 8.31$ J/(mol · K)
- Boltzmann's constant, $k_B = 1.38 \cdot 10^{-23}$ J/K
- Electron charge magnitude, $e = 1.60 \cdot 10^{-19}$ C
- 1 electron volt, 1 eV $= 1.60 \cdot 10^{-19}$ J
- Speed of light, $c = 3.00 \cdot 10^8$ m/s
- Universal Gravitational constant,
$$G = 6.67 \cdot 10^{-11}\ (\mathrm{N \cdot m^2})/\mathrm{kg}^2$$
- Solar Mass
$$M_\odot = 1.988 \cdot 10^{30}\ \mathrm{kg}$$
- Acceleration due to gravity, $g = 9.8$ m/s^2
- 1 unified atomic mass unit,
$$1\ \mathrm{u} = 1.66 \cdot 10^{-27}\ \mathrm{kg} = 931\ \mathrm{MeV/c^2}$$
- Planck's constant,
$$h = 6.63 \cdot 10^{-34}\ \mathrm{J \cdot s} = 4.41 \cdot 10^{-15}\ \mathrm{eV \cdot s}$$
- Permittivity of free space,
$$\epsilon_0 = 8.85 \cdot 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$$
- Coulomb's law constant,
$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \cdot 10^9\ (\mathrm{N \cdot m^2})/\mathrm{C^2}$$
- Permeability of free space,
$$\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{T \cdot m/A}$$
- Magnetic constant,
$$\frac{\mu_0}{4\pi} = 1 \cdot 10^{-7}\ (\mathrm{T \cdot m})/\mathrm{A}$$
- 1 atmospheric pressure,
$$1\ \mathrm{atm} = 1.01 \cdot 10^5\ \mathrm{N/m^2} = 1.01 \cdot 10^5\ \mathrm{Pa}$$
- Wien's displacement constant, $b = 2.9 \cdot 10^{-3}$ m · K
- Stefan-Boltzmann constant,
$$\sigma = 5.67 \cdot 10^{-8}\ \mathrm{W/m^2/K^4}$$

Problems

1 Short-Answer

1. Very Large Angle Pendulum

A simple pendulum of length l in a uniform gravitational field g is initially in stable equilibrium. It is given an impulse suddenly so that it has an angular velocity 4ω where $\omega := \sqrt{g/l}$. We find that the pendulum makes full turns with angular frequency $\beta\omega$ where β is slightly less than 4. Find $4-\beta$ to 1% accuracy. Note that a simple pendulum is essentially just a point mass on a string of length l.

2. Teo I

Upon the discovery of the exoplanet Teo I by the esteemed astronomer and astrophysicist Kai Wen, news sources around the world (that is to say, Earth) began buzzing with excitement due to the remarkable similarities Teo I shared with planet Earth. "From our readings, it appears that not only does Teo I share several physical similarities with Earth, with identical surface gravity, atmospheric composition, average temperature and size, but also *cultural* similarities!". Indeed, the residents of Teo I engage in many activities that may be familiar to the residents of our lovely planet. For example, chess, boxing, chessboxing, large amounts of gambling and of course, physics olympiads. Kai Wen decided to immediately challenge the inhabitants of Teo I to a high stakes no limit Holdem (also known as poker) match, wagering his entire life savings (approximately five (5) Singaporean dollars). Unfortunately, soon after the events of this problem, the astronomers of Earth reclassified Teo I as uninhabited following numerous mysterious explosions detected in the planet's atmosphere.

In order to reach Teo I, Kai Wen designs a photon rocket, which has a total mass $M = 10^5$ kg and a payload with mass $m = 2000$kg. The rocket has been calibrated to run out of fuel upon reaching Teo I, after decelerating to rest. (Due to a slight miscalculation, Kai Wen forgot to reserve any fuel to return to Earth). Naturally, in order to ensure the terms of the match are fair, Kai Wen sends out a radio message detailing his challenge and urging the inhabitants of Teo I to begin training their best poker team. To place both teams on an even playing field, both teams must engage in the same amount of training. From prior research, it is known that the rate of increase in one's poker ability is directly proportional to the magnitude of the local gravitational acceleration. The radio message and the photon rocket are launched simultaneously as measured by an observer at infinity.

What is the maximum possible distance, in light years, between Teo I and Earth?

3. Kugel Fountain A Kugel fountain is a water feature that uses the phenomenon of hydroplaning to suspend large rocks above a thin film of water.

One such fountain has a flat base. A circular hole is drilled through the center of the fountain and fitted with a vertical pipe of radius $a = 4\,\text{cm}$. The opening of the pipe is flush with the base. Water flows out of the pipe at a volume flow rate $Q = 100\,\text{cm}^3/\text{s}$. A puck-shaped rock with radius $R = 120\,\text{cm}$ is placed on the fountain, with its flat side resting on the base and its center right above the pipe's center. The water film suspends it, and thickness of the water film between the rock and the base is $h = 0.8\,\text{mm}$. Find the weight of the rock W in Newtons to 5% accuracy.

Assume that the water beneath the rock is in steady-state laminar flow (although this is not very realistic due to the high Reynolds number of the flow). The viscosity of water at room temperature is $\mu = 8.9 \times 10^{-4}\,\mathrm{Pa\,s}$.

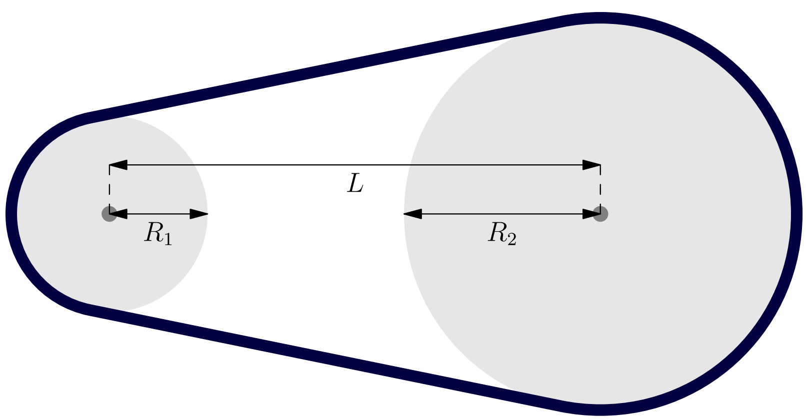
4. A Different Kind of Superfluid

Consider a very large tub of a superconducting fluid with negligible surface tension. The fluid has density $\rho_l = 670\ \mathrm{kg/m^3}$. A small magnet is slowly lowered to a height $h = 1$ m above the fluid surface. The magnet has net dipole moment of magnitude $m = 5929\ \mathrm{A \cdot m^2}$, oriented vertically. Find the magnitude of the maximum vertical deflection of the fluid surface (as compared to the height of the fluid surface very far from the magnet). Input your answer in micrometers.

5. Belt friction

Belts are sometimes used to "connect" the rotation of two wheels. This problem will explore the mechanics of such systems. There are two uniform cylinders, Cylinders 1 and 2. Cylinders 1 and 2 have masses $M_1 = 1\,\mathrm{kg}$ and $M_2 = 2\,\mathrm{kg}$, radii $R_1 = 0.2\,\mathrm{m}$ and $R_2 = 0.6\,\mathrm{m}$ respectively. The central axes of the cylinders are attached to separate horizontal frictionless axles a distance $L = 1.1\,\mathrm{m}$ apart such that the cylinders can freely rotate. A taut inelastic belt with a mass much less than the mass of the cylinders is wrapped around both cylinders. There is significant friction between the belt and the surface of the cylinders.

Cylinder 1 is quickly spun up to an angular speed of $\omega_0 = 20\,\mathrm{rad/s}$ (clockwise), fast enough that Cylinder 2 does not rotate while Cylinder 1 is being spun up. Then, the cylinders are left to rotate freely. After a time $t = 0.4\,\mathrm{s}$, both cylinders reach a constant angular velocity. Find the magnitude of the average vertical force exerted by the axle of Cylinder 1 on the cylinder (in Newtons), across the 0.4 seconds. Gravity is pointing downwards in the diagram provided.



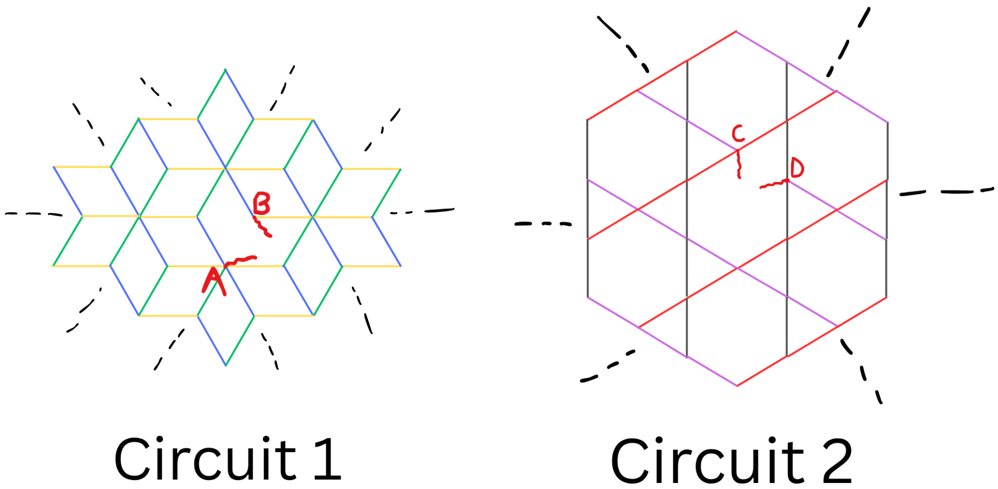
6. Stars and Deltas

Consider two infinite planar resistive networks.

Circuit 1. The network is obtained by tiling the plane with rhombi having interior angles 60° and 120°, as depicted. Each edge of the tiling is occupied by a resistor. Edges are partitioned into three classes by orientation, indicated by colour: yellow edges each carry resistance R_1, blue edges each carry resistance R_2, and green edges each carry resistance R_3. Let R_{AB} denote the equivalent resistance measured between the terminals A and B marked in the figure.

Circuit 2. The network is obtained by tiling the plane with regular hexagons and equilateral triangles (the tri-hexagonal tiling), as depicted, with a resistor on every edge. Grey edges each carry resistance

R_4, red edges each carry resistance R_5, and magenta edges each carry resistance R_6. Let R_{CD} denote the equivalent resistance between the terminals C and D marked in the figure.



In both networks, all mutually parallel edges (i.e. all edges of the same orientation) belong to the same colour class and hence carry equal resistance.

Given:$R_1 = 1\,\Omega$, $R_2 = 2\,\Omega$, $R_3 = 3\,\Omega$, $R_4 = 3\,\Omega$, $R_5 = 1.5\,\Omega$, $R_6 = 1\,\Omega$.

Determine the product $R_{AB} \times R_{CD}$, expressed in Ω^2.

7. Fridge Magnet

Note: This problem consists of two parts, but there is only one input. Hence, submit the **product** of both answers. For example, if your answer to part 1 is $2\,\mathrm{kA/m}$ and your answer to part 2 is $3\,\mathrm{kA/m}$, you should submit $2 \times 3 = 6$ as your final answer.

Iron is a ferromagnetic material which we assume to have a relative permeability $\mu_r = 1.0 \times 10^5$ and negligible hysteresis. Suppose we have a cylindrical neodymium magnet with mass density $\rho = 7500\,\mathrm{kg\,m^{-3}}$, radius R and length L. The neodymium magnet has a uniform magnetisation density M directed along its axis of rotational symmetry. We place the flat side of the magnet on the vertical wall of a fridge, which can be treated as a large cube of iron. The coefficient of static friction between the magnet and the fridge surface is $\mu_s = 0.3$ (not to be confused with the relative permeability defined earlier!). Assume that the magnetisation density of the neodymium magnets are independent of the magnetic field.

1. If $R = 10\,\mathrm{cm}$ and $L = 0.4\,\mathrm{cm}$, find the minimum value of M (in units of kA/m) such that the magnet remains stuck on the fridge in its original orientation.

2. If $R = 0.4\,\mathrm{cm}$ and $L = 10\,\mathrm{cm}$, find the minimum value of M (in units of kA/m) such that the magnet remains stuck on the fridge in its original orientation.

8. Fabric of Reality

Consider a weightless fabric with constant surface tension γ, initially stretched taut in the horizontal plane. The outer boundary of the fabric is a circle of radius b. A circular region of radius $a \ll b$ is carved out from the fabric, concentric with the outer boundary. A ring with weight $W < 2\pi a\gamma$ uniformly distributed along its length is then attached along the inner circular boundary. This causes the fabric to sag under the action of the mass.

Particles with negligible mass can perform circular orbits around the center of the fabric. When $\frac{W}{\gamma a} < \xi$ holds, all such circular orbits are stable. Determine the numerical value of ξ.

9. Beam Splitter

Zeus, an aspiring optics enthusiast, fires a laser producing a beam of unpolarized light. Being somewhat careless with expensive equipment, he manages to trap this beam inside a glass slab of refractive index $n = 4$, length l, and breadth $b = 2\,\mathrm{m}$, with the beam propagating at an angle $\theta = 30^\circ$ to the normal. Zeus positions a camera at the far end inside the slab.

The camera can distinguish between the two linearly polarized components if the separation between the centers of the two beams is $x = 1\,\mathrm{mm}$. In order for the two components to be distinguishable, what is the minimum length ℓ (in meters) of the glass slab? The light has a vacuum wavelength $\lambda = 633\,\mathrm{nm}$, and the slab is surrounded by air.

Neglect the diffraction effects of the laser.

10. Countless Collisions

This problem is one-dimensional, with all objects constrained to the x-axis. Consider a block of mass M with initial velocity $-v_0$ and position $x = L$ at $t = 0$. A dispenser is positioned at $x = 0$, given an initial velocity v_1, and uniformly releases small balls of mass m and initial velocity u relative to the lab frame in time intervals of k starting at $t = 0$. If all collisions are elastic, find the minimum distance between the dispenser and block after a very long time has elapsed. Neglect all collisions with the dispenser and assume that the dispenser always moves at the same velocity. Report the answer in meters.

Use the following parameters: $M = 59290$ kg, $v_0 = 5.929$ m/s, $v_1 = 2.34$ m/s, $L = 5929$ m, $m = 0.5929$ kg, $k = 0.005929$ s, $u = 5.929$ m/s.

11. Diffractive Cryptography

The five images below shows five diffraction patterns. The central image is a diffraction pattern from an aperture which resembles one mathematical binary operation $(+, -, \times, \div)$, while the rest represents the English alphabet. The possible alphabetical apertures are the following letters, with the corresponding assigned digits: $A = 01$,$B = 02$, ..., $Z = 26$. The font of the aperture is sans serif, that is, written such that the edges of the letter is a simple straight edge. To encrypt the message, the corresponding digits of each apertures are concatenated, then operated by the binary operation. Determine the result of the encrypted message up to three significant figures. (e.g. if the apertures are AC$\times$KM, it represents 0103×1113).

Do not solve this by using external tools to apply the Fourier Transform.

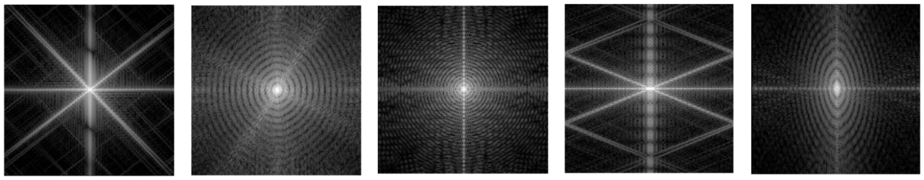


Figure 1: Diffraction patterns.

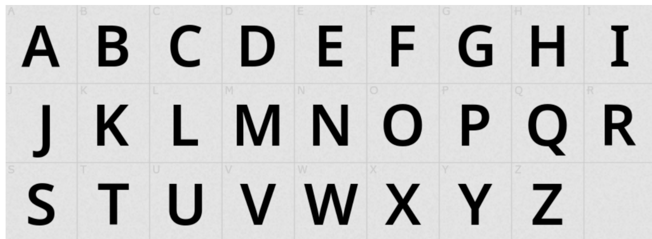


Figure 2: Example font used in the message.

12. Compressing Light

A perfectly insulating rigid cylinder is closed at one end and sealed by a movable piston at the other. The cylinder contains a photon gas at initial temperature $T = 1000$ K in a complete vacuum. The walls and piston are purely reflecting. Initially, the cylinder has a length L and cross sectional area $A = 5.929$ m^2. The piston is slowly compressed until the final length of the cylinder is $L_f = L/8$. Find the pressure exerted by the gas on the face of the piston. Give your answer in Pa.

13. The Odyssey

Achilles has sealed, opaque container holding one mole of free neutrons together with one mole of a hypothetical particle called the "Odysseon," whose rest mass is $m_{Od} = 0.511$ MeV$/c^2$. The two species inside are in mutual thermal equilibrium at a temperature $T = 1.4 \times 10^{10}$ K.

He then pokes a tiny hole in the container's wall, allowing particles of both types to effuse out. As each particle escapes, Achilles records its kinetic energy, and in this way builds up a separate kinetic-energy distribution (spectrum) for each species.

Odysseus, in his wisdom, advises Achilles against analyzing two separate spectra. Achilles, however, trusts his own approach and presses on — computing the root-mean-square (RMS) fluctuation in kinetic energy of the effusing particles, once for the neutrons and once for the Odysseons.

Determine the ratio of these RMS kinetic-energy spreads(standard deviation),

$$\rho = \frac{\sigma_{KE(Od)}}{\sigma_{KE(n)}}.$$

Assumptions: the effusion process is quasi-static, both gases behave ideally, and thermal radiation inside the chamber may be ignored.

14. Pursuit of Happiness?

Consider a standard two-dimensional flat Cartesian plane somewhere deep in outer space (called Tristanland). An overworked member of the OPhO committee is standing at rest at the origin. Suddenly, they see an Amazingly Gracious Insect (AGI) at rest at $(0, L_0 = 10^9\ \mathrm{m})$. At $t = 0$, the committee member begins chasing the AGI in their vision; they run at a constant speed of $v_C = 10^8\ \mathrm{m/s}$, while the elusive AGI begins running at the same speed in the positive x direction as soon as it sees the committee member coming. After a long time, there is a constant distance d between the committee member and the AGI; find d in meters.

15. Aberration A convex lens is formed by a flat circular surface of diameter $2r_0$ and a spherical surface with radius of curvature R, as shown in the figure. It has a refractive index $n = 1.5$.

Light of uniform intensity $I = 30\mathrm{W\,m^{-2}}$ is incident on the flat surface parallel to the optical axis. The entire lens is illuminated. The intensity pattern produced on the focal plane is shown in the image below. (The y-axis is in units of $\mathrm{kW\,m^{-2}}$ and the x-axis is in units of mm.)

You should be able to reverse engineer the radius of curvature R and also the diameter of the lens $2r_0$ from the plot. Find their product $2Rr_0$ in units of m^2, correct to 5%. Ignore diffraction effects.

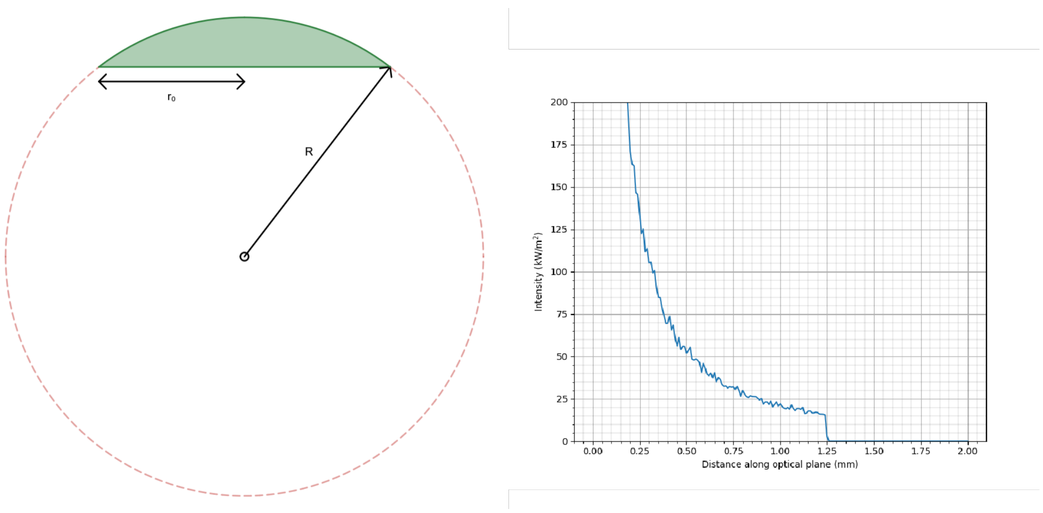
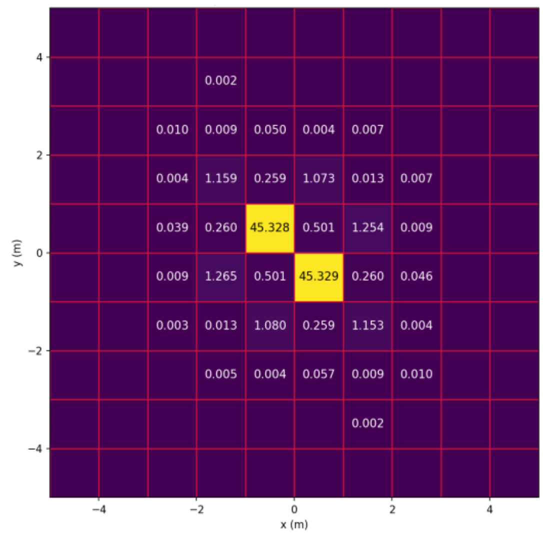


Figure 3: Left: Shape of the lens. Right: Graph of intensity.

16. Bee greed

A large number of particles with $q/m = 0.5$ C/kg that do not interact with each other have been launched with an uniform, isotropic angular distribution spanning on the xy plane from the origin with unknown speed v. Space around that point is divided along a 2d grid analogous to a chessboard/checkerboard pattern with side length a, squares of one "color" (Not the one in the image) have a magnetic field $\vec{B}$ with an unknown magnitude along the $\hat{z}$ axis, and those of the other one have no magnetic field inside them. Find the radius of curvature of the particles' motion in the B field using the following heatmap of squares in which they were detected between 55 and 65 seconds. (The number embedded in each square is the average percentage of particles that ended up there)



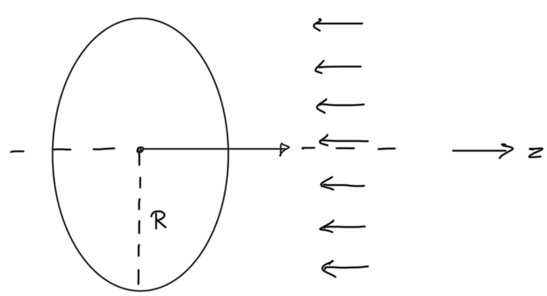
17. Not Quite Now

A plane electromagnetic wave propagates in vacuum along the $-Z$-direction. In the lab frame, its intensity profile at the initial time $t = 0$ can be approximated as:

$$I_{\text{lab}}(x, y, z, t = 0) = I_0 \left(1 + \frac{z}{L}\right),$$

for (x, y, z) over scales $\gg R$, and the profile thereafter propagates rigidly at speed c. An object which is a sphere of radius R in its own rest frame is constrained by an external force to move along $+Z$ with constant relativistic speed v; its surface has absorptivity α and reflectivity $r = 1-\alpha$, with reflection taken to be specular. Note that r and α are constant across the surface, independent of wave polarization or the angle of incidence.

At every instant in the sphere's rest frame, only a fraction η of the absorbed energy is retained by the sphere, the remainder being re-emitted isotropically in that frame.

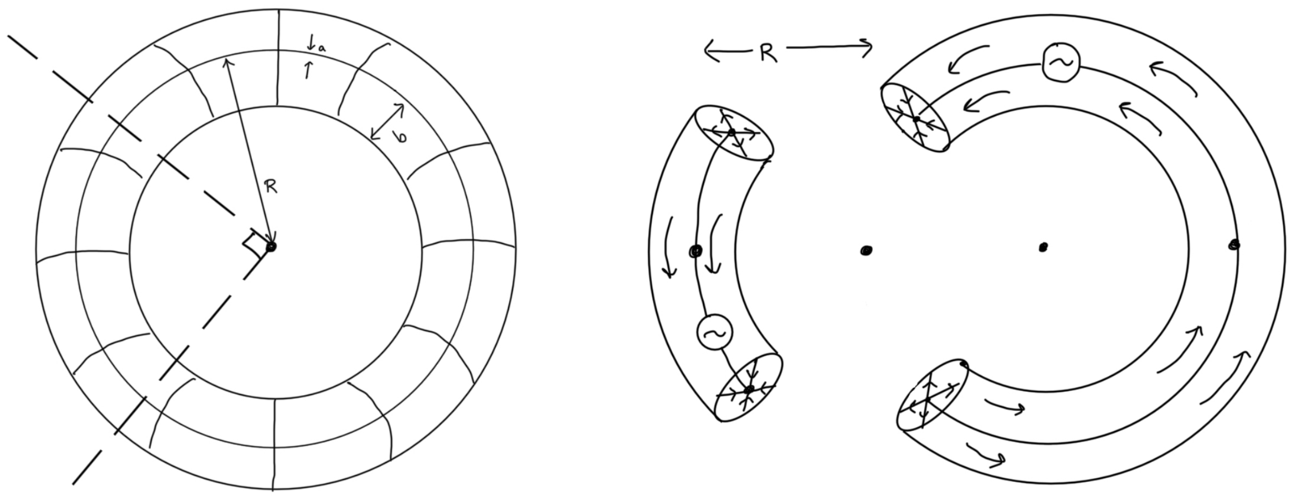


Assuming the sphere's size is **NOT** negligible compared to the length scale L, find the force $F_{rad}(t)$ applied by the radiation on the sphere as felt in the lab frame, if the sphere's center starts from the origin at the initial time $t = 0$.

Numerical data: Take $I_0 = 300\,\mathrm{W\,m^{-2}}$, $L = 3.0\,\mathrm{m}$, $R = 0.30\,\mathrm{m}$, $v = \frac{3}{5}c$, $\alpha = \frac{2}{5}$, $\eta = \frac{1}{2}$ (so $r = \frac{3}{5}$), and evaluate $F_{\text{rad}}(t)$ at $t = 2.50\,\mathrm{ns}$. Evaluate in Newtons, and use a negative sign if force is along $-Z$ and vice versa.

18. Farampere

Consider a hollow metallic ring torus of mean radius R and tube thickness $2b$, with $b \ll R$. A 90° wedge of the torus is cut out and displaced outward, along its own line of symmetry, by a distance R (see the figures below). We call the displaced quarter A and the remaining three-quarter arc B.



Each piece is wired as a bent coaxial line. A uniform current runs along the toroidal (outer) surface in the directions shown, and at each open end radial spokes hand it off to an inner wire of radius a running along the axial circle, so that within both A and B the current closes on itself. An external source drives the current at a common time-rate $\dot{I}$. Find the interaction force between A and B arising from the changing current.

For a numerical estimate, take $a = 1.0\,\mathrm{mm}$, $b = 5.0\,\mathrm{cm}$, $R = 10\,\mathrm{m}$, $I = 0\,\mathrm{A}$, and $\dot{I} = 10^8\,\mathrm{A\ s^{-1}}$; Use a negative sign if the force is attractive (the pieces pull together) and a positive sign if it is repulsive.

19. Whiplash

A perfectly flexible, inextensible rope of length $L = 100$ m and linear mass density $\mu = 0.1$ kg m^{-1} is attached to the end of a rigid rod of length $h = 0.3$ m. The other end of the rod is kept fixed at the origin. The rod is suddenly rotated by 180 degrees at an angular velocity $\omega = 30$ rad/s. The initial shape of the rope is an arc of radius $R = 250$ m, and the rope makes a right angle with the rod where they are attached. Find the total kinetic energy of the rope, in joules. Ignore air resistance and gravity.

2 Long-Answer

20. Spherical Membrane

An ideal gas consisting of $2N$ classical particles of mass m is confined within an harmonic isotropic potential.

$$V(r) = kr^2/2$$

The system is kept at a constant temperature T by a thermal bath. A massless, perfectly flexible and perfectly conducting spherical membrane is placed concentrically with the origin. Let the radius of the membrane be R. For all parts, assume $N \gg 1$. When possible, give your answer in terms of $\lambda = \sqrt{k_B T/k}$.

Initially, the membrane is completely impermeable to the gas particles, and acts as a movable spherical piston. It partitions the gas into two subsystems, an "inner" gas containing N particles with $r < R$ and an "outer" gas containing N particles in the region $r > R$. The membrane can freely expand or contract. In the thermodynamic limit ($N \to \infty$), the membrane settles at an equilibrium radius R_0.

(a) Given $R_0 = \alpha\lambda$, find the value of α. [1.5 pts]

As N is finite, the membrane radius R actually undergoes tiny fluctuations around R_0.

(b) Derive an expression for the mean-square fluctuation $\langle (R - R_0)^2 \rangle$ in terms of N, λ and α. [2.5 pts]

When the fluctuations of the membrane are ignored, the system has a heat capacity C_0. However, when the membrane is free to fluctuate the system has a different heat capacity C_1.

(c) Calculate the difference $\Delta C = C_1 - C_0$. [0.5 pts]

Suppose the membrane is now semi-permeable, allowing gas particles to pass through it only if the radial velocity satisfies $mv_r^2/2 > \epsilon_0$, where ϵ_0 is a fixed energy threshold.

(d) Find all macroscopic equilibrium positions R_1 of the membrane. [1.0 pts]

Now, consider fixing the membrane at R_0. Let N_i denote the number of particles with $r < R_0$.

(e) Find the mean square fluctuation $\langle (N_i - N)^2 \rangle$. [1.5 pts]

We now once again consider the non-permeable membrane. Suppose the system is initially in thermodynamic and mechanical equilibrium. At time $t = 0$, the stiffness of the harmonic trap is suddenly doubled from $k \to 2k$.

(f) Find the net force acting on the membrane at $t = 0$. [0.5 pts]

Suppose the membrane now has a constant surface tension $\sigma > 0$, with $\sigma\lambda^2 \ll Nk_B T$. The trap stiffness is restored to its original value, and the system is allowed to reach equilibrium.

(g) Estimate the change in the equilibrium radius ΔR to first order. [2.5 pts]

21. Teo II

Kai Wen has discovered another planet: Teo II, a small, dense core with mass M surrounded by an atmosphere with negligible mass. The atmosphere is isothermal with temperature T, and is composed of two gases with equal adiabatic constants but different molecular masses $\alpha m, (1-\alpha)m$ where $\alpha < 1/2$. Define the dimensionless radial coordinate $\eta = \dfrac{rkT}{GMm}$, and let the mole fraction of the lighter gas be $\chi(\eta)$.

Kai Wen is still angry about his decisive poker loss to the inhabitants of Teo I. He detonates a large bomb at radius η_0, sending sound waves through the atmosphere. After a long time, he notices that the sound intensity is large at radius η_0.

(a) Find η_0 in terms of $\chi_0 = \chi(\eta_0)$. [3.5 pts]

(b) What are the possible values for η_0, if α is fixed but the atmospheric composition can vary? [3.5 pts]

(c) Inspecting more closely, he sees that in general, the locations of locally maximum intensity on the sphere of radius η_0 form rings. What is the smallest possible η_0 such that there are *no* rings? [3.0 pts]

22. Einstein Ring

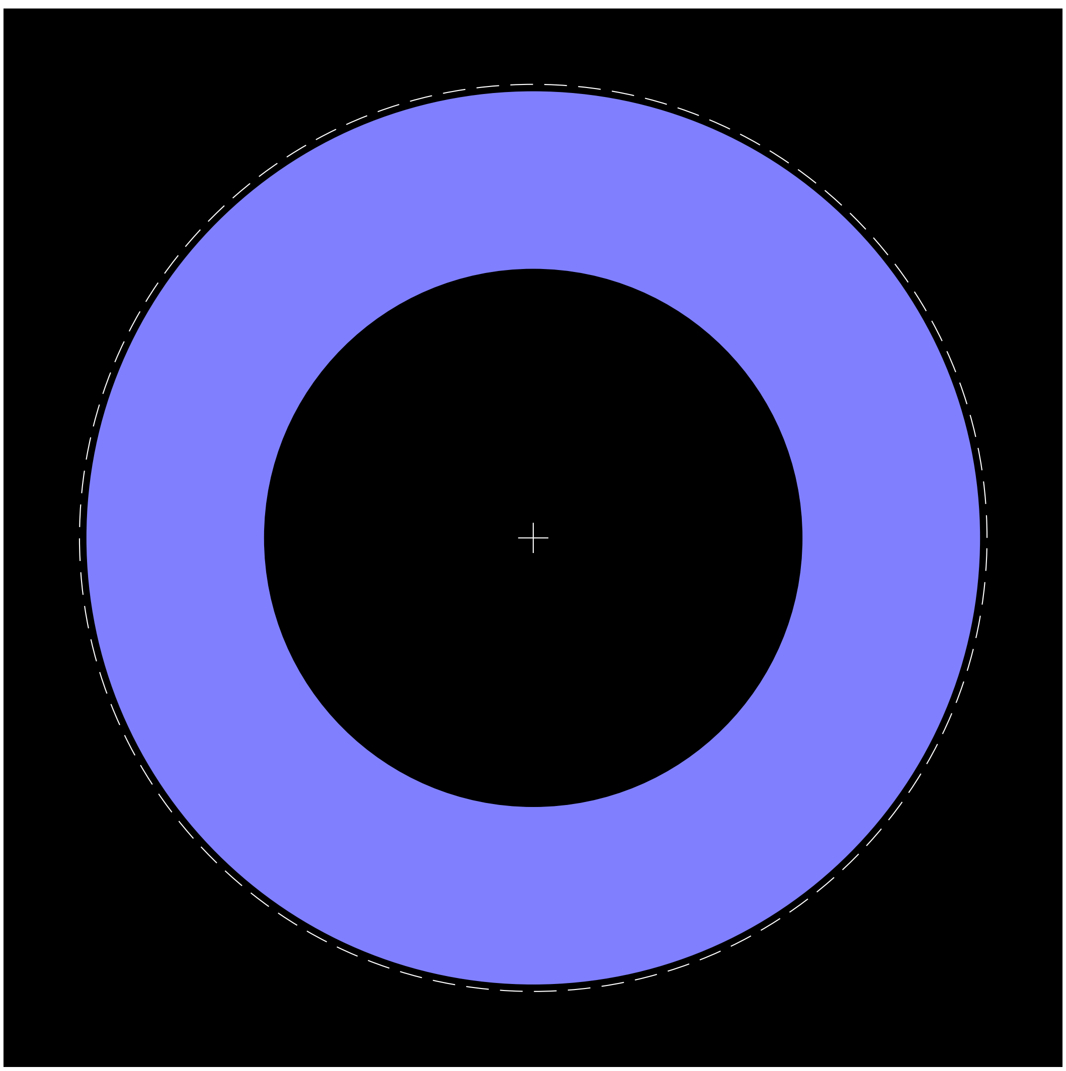
A transparent unit sphere has a small hemispherical indent in its surface, located at $\hat{z}$. UV light shines on the indent from the $\hat{z}$ direction. The sphere is doped with fluorescent dye that has a chance to absorb UV light and reemit it as blue light in a random direction. The concentration of dye is small, so a UV photon typically travels a very long distance before being absorbed. Assume that light is either totally reflected or transmitted at interfaces.

A camera far away from the sphere in the $\hat{z}$ direction takes the following image; the center and boundary of the sphere are marked. If the sphere's indices of refraction for blue and UV light are n_b and n_u respectively:

(a) Find n_b. [6.0 pts]

(b) Find n_u. [5.0 pts]

(c) For which values (n_b, n_u) does a similar ring appear? [4.0 pts]



3 Simulation Lab

23. Spinny Spin Spin

It is well known that the spin of an electron is related to its magnetic moment by $\boldsymbol{m} = -g\mu_B \boldsymbol{S}$, where μ_B is the Bohr magneton, g is a dimensionless factor that depends on the electron's environment, and $\boldsymbol{S}$ is a vector of magnitude $\frac{1}{2}$. The angular momentum of the electron is given by $\hbar\boldsymbol{S}$.

Now, an ensemble of $N = 100000$ electron spins are distributed roughly evenly along a rod of length 2. The rod points in the x direction. The left end of the rod is given the coordinate $\boldsymbol{\xi} = -1$ and the right end of the rod is given the coordinate $\boldsymbol{\xi} = 1$. The magnetic field along the rod points in the z direction and is given by $B_z = B_0 + B_1\xi$. It is known that B_0 is on the order of $0.01T$ and $B_1 \ll B_0$. You can also add a bias magnetic field $\boldsymbol{B} = b\hat{z}$ which is uniform, but due to budget constraints this magnetic field must be limited to a magnitude of $0.01T$.

The spins are placed in a thermal bath and allowed to thermalise. In this initial stage, the spin will either align with the magnetic field or antiparallel to it according to the Boltzmann distribution. At time $t = 0$, a strong magnetic field pulse is applied to the system to rotate all spins 90° anticlockwise about the y axis. You can then set a sensor to detect the total magnetisation around the x and y axes at certain times (for convenience it will be given in units of μ_B), but remember these measurements are subject to noise! Also, a protection circuit is in place to prevent the initial magnetic field pulse from destroying the measurement circuit, so nothing will be recorded before $t = 100$ns. After $t = 10000$ns the measurement circuit runs out of memory and nothing will be recorded either.

Your goal is to find g, B_0 to 5% accuracy, as well as B_1 to 15% accuracy. Give the magnetic fields in units of Tesla. You may assume the spins do not interact with the environment or with each other after $t = 0$.